MATHEMATICS 2001
GROUPWORK DUE NOVEMBER 11

Tasks

(1) If you were elected presenter but did not actually present in class, that doesn't count as having presented. Check D2L. If you have anyone in your group who needs to present, they can email me to make sure it happens.

(2) **Main Task 1: Take up homework done so far.** As in previous weeks.

(3) **Main Task 2: Group Homework.**

  (a) Consider the set S of **nonzero** vectors in $\mathbb{R}^2$. We will define a relation on this set, by saying two vectors v_1 and v_2 are equivalent (write $v_1 \sim v_2$) whenever one is a scalar multiple of the other. Prove that this relation is an equivalence relation.

  (b) Explain why this relation would not be an equivalence relation if the set S included the zero vector.

  (c) Any equivalence relation $\sim$ on S breaks up S into *equivalence classes*, subsets of things which are all equivalent to one another. Formally speaking, the equivalence class of an element a is the set

$$[a] = \{b \in S : a \sim b\}.$$

  Give examples of the equivalence classes for the relation on $\mathbb{R}^2$ in the last problem. Explain why the equivalence classes of the last problem should be thought of as lines in the plane.

  (d) Consider the set $S = \mathbb{Z} \times (\mathbb{Z} - \{0\})$. Define a relation on this set, by saying two pairs (a, b) and (c, d) are equivalent if $ad = bc$. Prove that this is an equivalence relation.

  (e) Explain why the equivalence classes of this last relation should be thought of as the rational numbers.

  (f) Prove that there exists a relation on the integers which is symmetric but not reflexive.

  (g) Disprove the following statement: Every relation on the integers is transitive. (To **disprove** something means to prove its negation.)

(4) Fill out your groupwork report and have everyone sign. **This is due in class.**

(5) The scribe will prepare a PDF of your proofs to hand in on D2L. **I appreciate getting these early on Friday so I can look through them.**