MATHEMATICS 2001
GROUPWORK DUE DECEMBER 2

Tasks

(1) If you were elected presenter but did not actually present in class, that doesn't count as having presented. Check D2L. If you have anyone in your group who needs to present, they can email me to make sure it happens. This may be the last chance! (Not sure we'll do groupwork on the very last week. However, there's an option (see below) for two people to present as a team.)

(2) **Main Task 1: Take up homework done so far.** As in previous weeks.

(3) **Main Task 2: Group Homework.**

(a) Prove that $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = x + 3$ is bijective. (Hint: this involves two sub-tasks, surjectivity and injectivity.)

(b) Consider this Theorem: *Suppose n pigeons are put into m holes. If $n > m$, then, no matter how this is done, at least one hole has at least two pigeons in it.*

(c) After considering it (that means trying some examples etc.), prove it. Hint: use contradiction.

(d) If you have n identical black socks and n identical white socks in a drawer, how many do you need to pull out with your eyes closed before you are sure you have a pair? Explain by reference to pigeonhole principle. What are the pigeons and what are the holes?

(e) Are there at least two people in New York city who have the exact same number of hairs on their heads? This involves pigeonhole and a bit of Fermi estimation – you may want to examine each others' heads.

(f) How many numbers do you need to pick from the set $\{1, \ldots, 8\}$ until you are certain you have some pair that adds to 9? (e.g. $1 + 8 = 9$) Explain with Pigeonhole. What are the pigeons and what are the holes?

(g) Consider this Theorem: *Suppose $f : A \to B$ is a function, where A and B are finite sets. Suppose $|A| > |B|$. Then f cannot be injective.* Explain why this is the same as the Pigeonhole Principle above. What are the pigeons and what are the holes?

(h) Prove this variation: *Suppose infinitely many pigeons are put into finitely many holes. Then at least one hole has infinitely many pigeons in it.*

(i) Prove the following: Let α be any real number with an infinite decimal expansion. Then there exists some 10-digit sequence which appears infinitely often in its expansion.

(j) Learn to perform this card trick and perform it in class (it requires two people, who can both get credit for presenting) in front of your classmates. You need a standard 52-card deck. A magician asks an audience member to pick five cards, which are not shown to the magician. The magician's accomplice looks at the cards, picks four of the cards and shows these four to the magician in an order of his choosing. The magician then correctly guesses the fifth card. The pigeonhole principle guarantees that this trick is possible; can you figure out how to do it? (Hint: first figure out why pigonehole principle guarantees it is possible.)

(4) Fill out your groupwork report and have everyone sign. **This is due in class.**

(5) The scribe will prepare a PDF of your proofs to hand in on D2L. **I appreciate getting these early on Friday so I can look through them.**