The game of SET and its geometric generalizations

Celebration of the Mind

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University of Colorado, Boulder

October 21, 2015

Each card has four characteristics:

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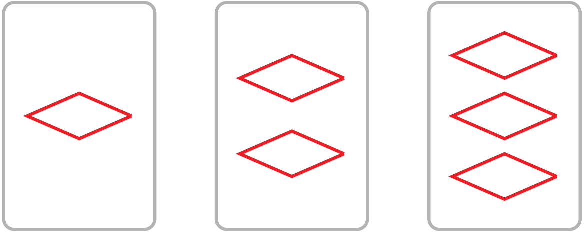
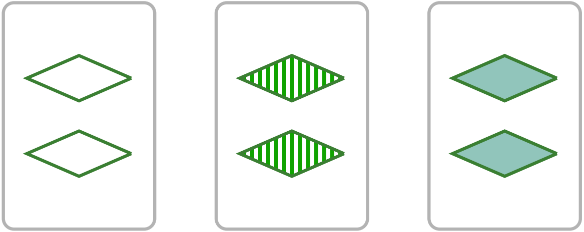
number



Each card has four characteristics:

number

shading



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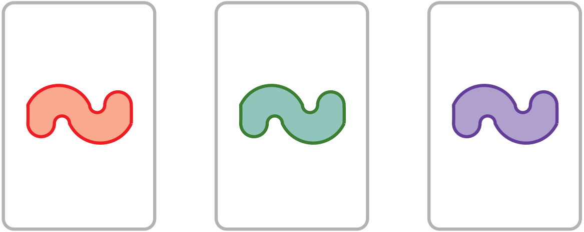
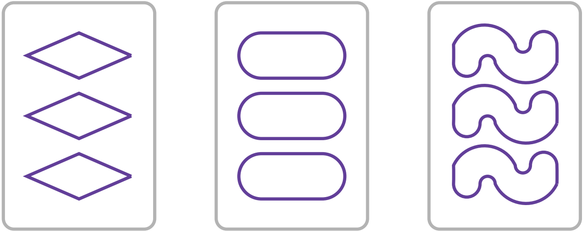


Each card has four characteristics:

number
shading
color
shape



Each card has four characteristics:

number

shading

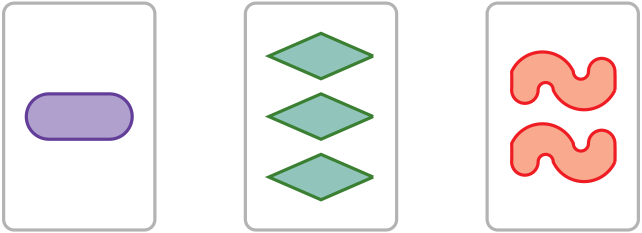
color

shape

Three possibilities for each of four characteristics gives a total of $3 \times 3 \times 3 \times 3 = 81$ cards.

A SET is a collection of 3 cards such that in each characteristic all 3 cards are either the same or different.

A SET:



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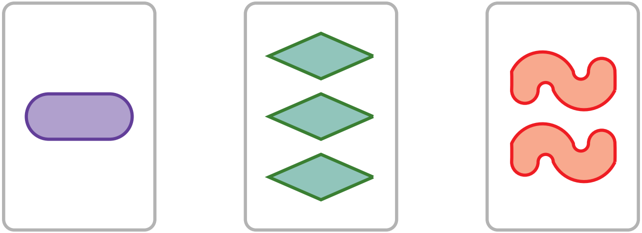
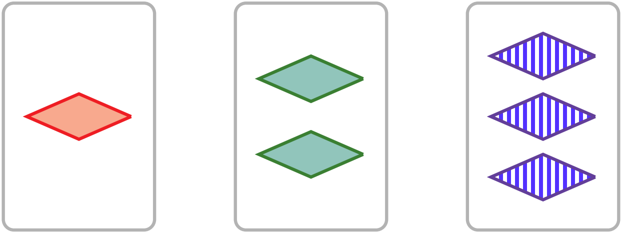


The cards have different numbers, the same shading, different colors, and different shapes.

A SET is a collection of 3 cards such that in each characteristic all 3 cards are either the same or different.

Not a SET:



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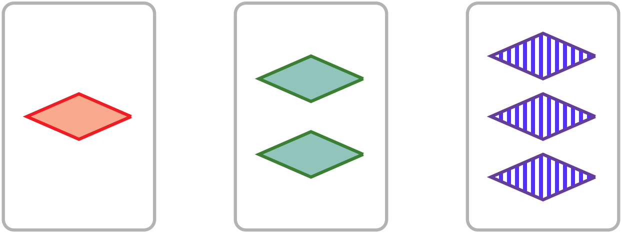
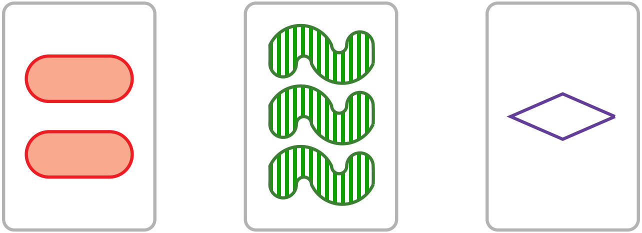


The cards have different numbers, different colors, and the same shape. But the first two cards have the same shading while the third is different.

A SET is a collection of 3 cards such that in each characteristic all 3 cards are either the same or different.

Is this a SET?



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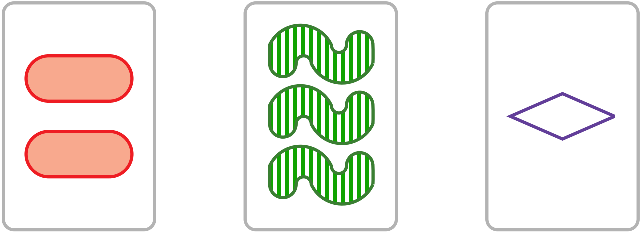


YES! The cards have different numbers, different shading, different colors, and different shapes.

How many SETs are there?

How many SETs are there?

There are 81 possibilities for the first card...

$$81$$

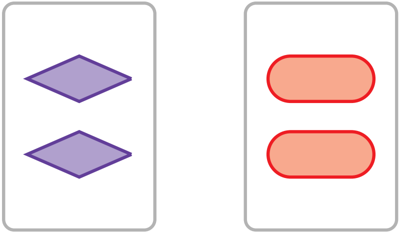
How many SETs are there?

There are 81 possibilities for the first card...
There are 80 possibilities for the second card...

$$81 \times 80$$

The last card in a SET is determined by the others

Suppose we start with any two cards:



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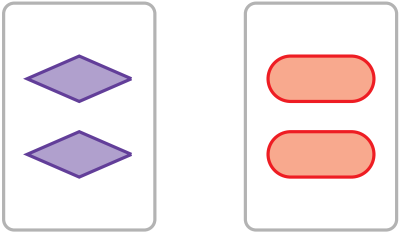

To make a SET, the third card must have:

number 2

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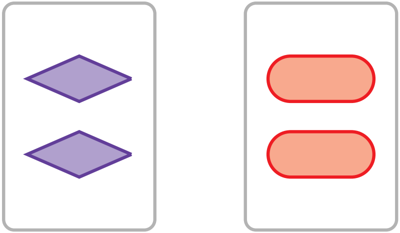

To make a SET, the third card must have:

number 2

color green

The last card in a SET is determined by the others

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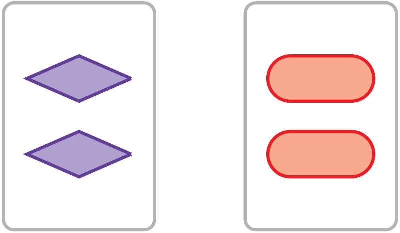

To make a SET, the third card must have:

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color green

shading solid

The last card in a SET is determined by the others

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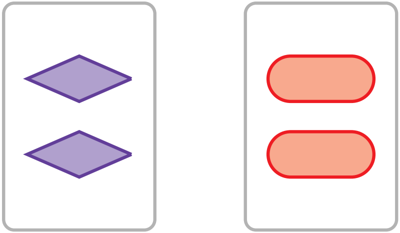

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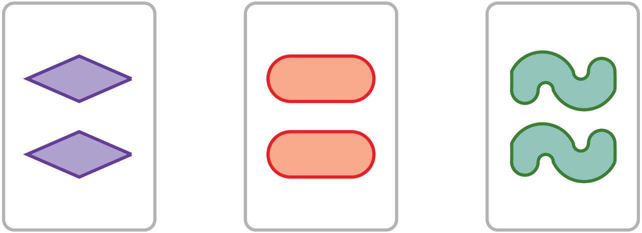

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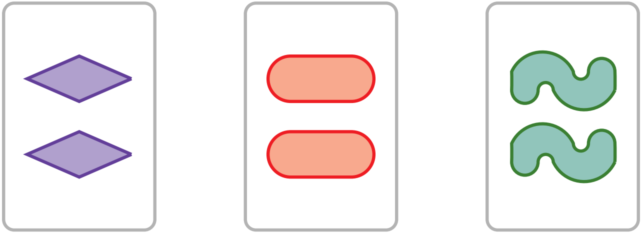

If the first two cards are the same in some characteristic, then the third card is also the same in that characteristic.

The last card in a SET is determined by the others

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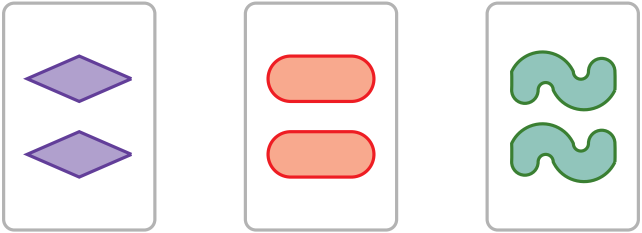

If the first two cards are the same in some characteristic, then the third card is also the same in that characteristic.

If the first two cards are different in some characteristic, then the third card will have the only remaining possibility for that characteristic.

How many SETs are there?

There are 81 possibilities for the first card...
There are 80 possibilities for the second card...
The third card is determined by the first two.

$$81 \times 80 \times 1$$

How many SETs are there?

There are 81 possibilities for the first card...
There are 80 possibilities for the second card...
The third card is determined by the first two.
But there are 6 different ways we could have chosen to order the cards.

$$\frac{81 \times 80}{6}$$

How many SETs are there?

There are 81 possibilities for the first card...
There are 80 possibilities for the second card...
The third card is determined by the first two.
But there are 6 different ways we could have chosen to order the cards.

$$\frac{81 \times 80}{6} = 1080$$

An unusual coincidence...

SET

For every 2 distinct cards there is a unique SET containing them

Geometry

An unusual coincidence...

SET	Geometry
For every 2 distinct cards there is a unique SET containing them	For every 2 distinct points there is a unique line containing them

SuperSETs

A (nonempty) collection of cards $\mathcal{S}$ is called a *super*SET if, for every pair of distinct cards P and Q in $\mathcal{S}$, the unique set containing P and Q is also in $\mathcal{S}$.

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3 cards—ordinary sets

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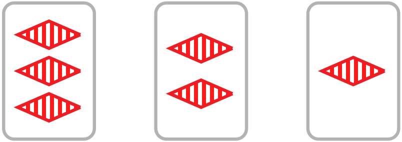
1 card—the degenerate case

3 cards—ordinary sets

81 cards—the whole deck!

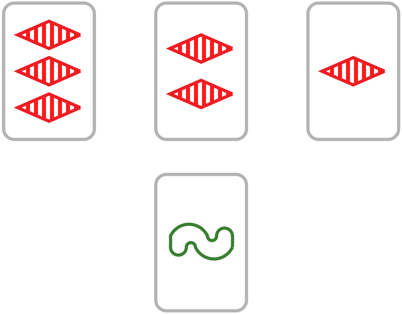
Building a superSET:

Start with an ordinary SET:



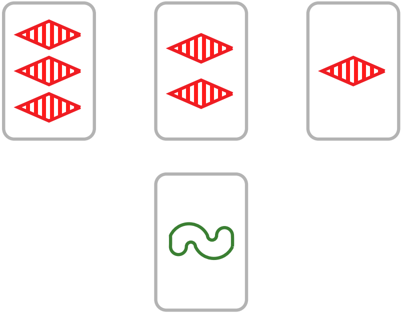
Building a superSET:

Add one more card:



Building a superSET:

Then we'll need to complete some SETs:



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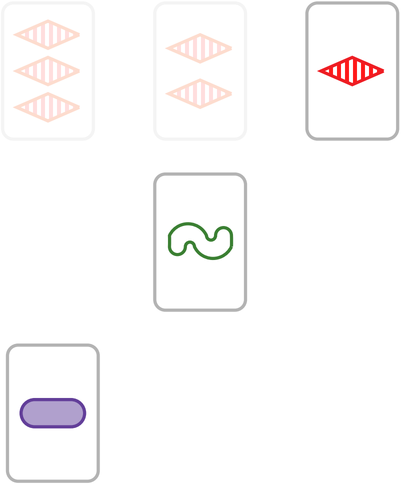

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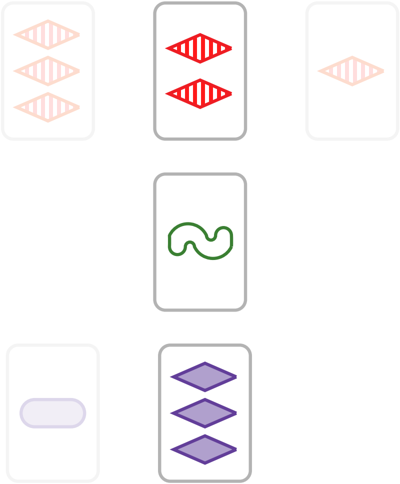

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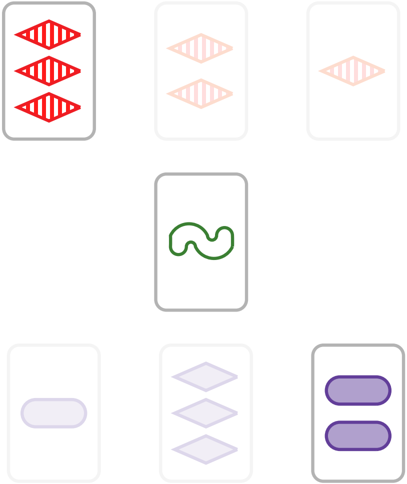
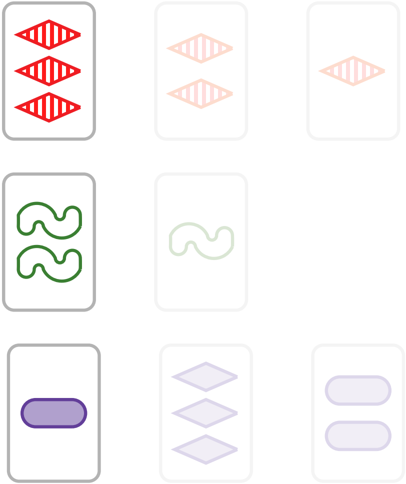


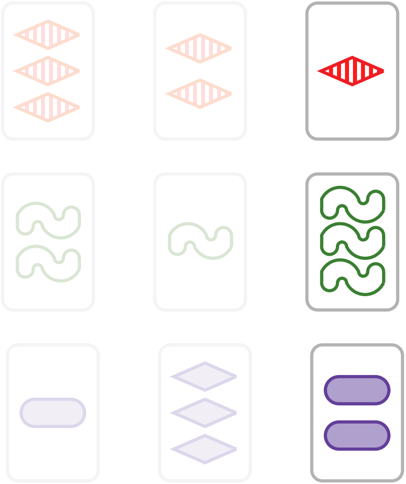
Building a superSET:

But we still don't have a superSET:



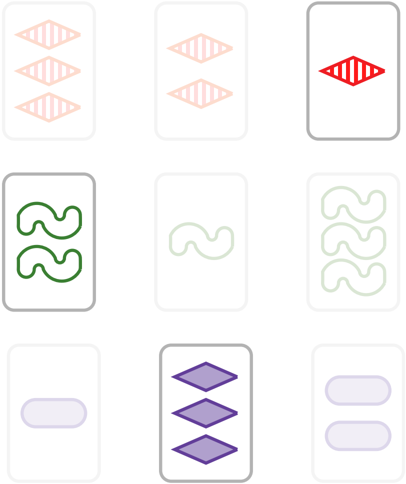
Building a superSET:

At last!



Building a superSET:

But there is still a lot of checking to make sure...



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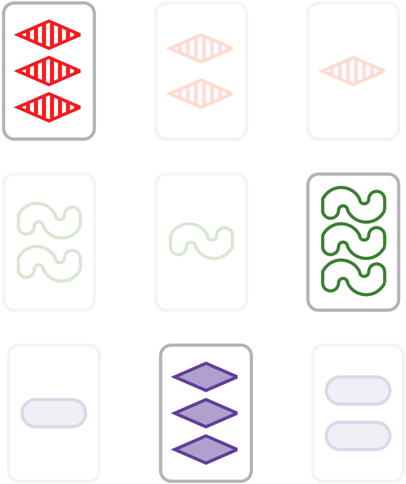

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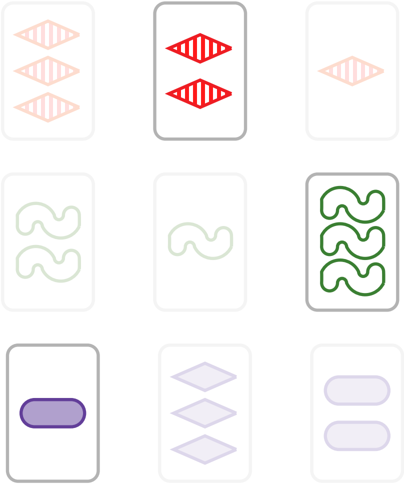

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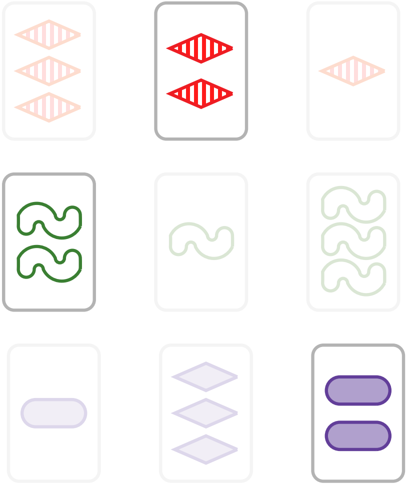

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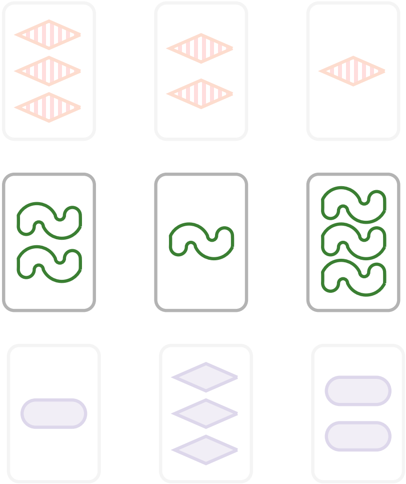

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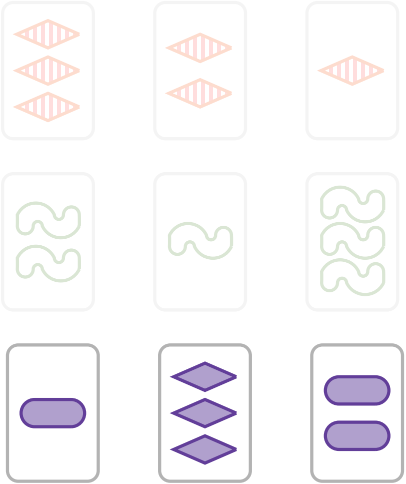

SuperSETs

A (nonempty) collection of cards $\mathcal{S}$ is called a *super*SET if, for every pair of distinct cards P and Q in $\mathcal{S}$, the unique set containing P and Q is also in $\mathcal{S}$.

SuperSETS can have

1 card—the degenerate case

3 cards—ordinary sets

9 cards

81 cards—the whole deck!

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SuperSETS can have

1 card—the degenerate case

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9 cards

27 cards

81 cards—the whole deck!

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SuperSETS can have

$1 = 3^0$	called 0-SETs
$3 = 3^1$	called 1-SETs
$9 = 3^2$	called 2-SETs
$27 = 3^3$	called 3-SETs
$81 = 3^4$	called 4-SETs

An unusual coincidence...

SET

For every 2 distinct cards there is a unique SET containing them

Geometry

For every 2 distinct points there is a unique line containing them

An unusual coincidence...

SET

For every 2 distinct cards there is a unique 1-SET containing them

Geometry

For every 2 distinct points there is a unique line containing them

An unusualer coincidence...

SET	Geometry
For every 2 distinct cards there is a unique 1-SET containing them	For every 2 distinct points there is a unique line containing them
For every 3 cards not contained in a 1-SET there is a unique 2-SET containing them	For every 3 points not contained in a line there is a unique plane containing them

An unusualer coincidence...

SET

For every 2 distinct cards there is a unique 1-SET containing them

For every 3 cards not contained in a 1-SET there is a unique 2-SET containing them

For every 4 cards not contained in a 2-SET there is a unique 3-SET containing them

Geometry

For every 2 distinct points there is a unique line containing them

For every 3 points not contained in a line there is a unique plane containing them

For every 4 points not contained in a plane there is a unique 3-plane containing them

An unusualer coincidence...

SET	Geometry
For every 2 cards not contained in a 0-SET there is a unique 1-SET containing them	For every 2 points not contained in a 0-plane there is a unique line containing them
For every 3 cards not contained in a 1-SET there is a unique 2-SET containing them	For every 3 points not contained in a 1-plane there is a unique 2-plane containing them
For every 4 cards not contained in a 2-SET there is a unique 3-SET containing them	For every 4 points not contained in a 2-plane there is a unique 3-plane containing them

An unusualer coincidence...

SET	Geometry
For every 2 cards not contained in a 0-SET there is a unique 1-SET containing them	For every 2 points not contained in a 0-plane there is a unique line containing them
For every 3 cards not contained in a 1-SET there is a unique 2-SET containing them	For every 3 points not contained in a 1-plane there is a unique 2-plane containing them
For every 4 cards not contained in a 2-SET there is a unique 3-SET containing them	For every 4 points not contained in a 2-plane there is a unique 3-plane containing them

and so on...

An unusualer coincidence...

SET

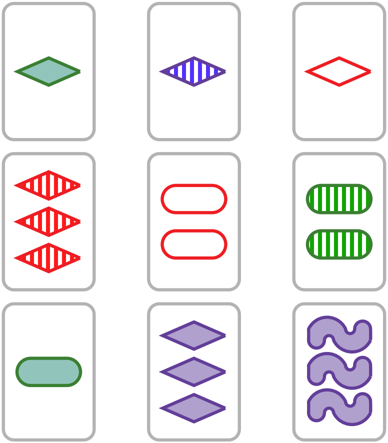
For every $n+1$ cards not contained in an $(n-1)$-SET there is a unique n-SET containing them

Geometry

Fr every $n+1$ points not contained in an $(n-1)$-plane there is a unique n-plane containing them

The game of CAP

Can you find 4 cards that lie in the same 2-SET?



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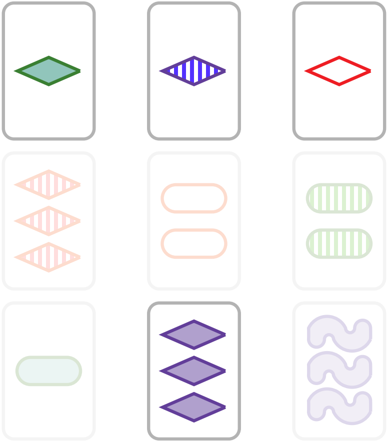

One way is to find a 1-SET and then add any card to it.

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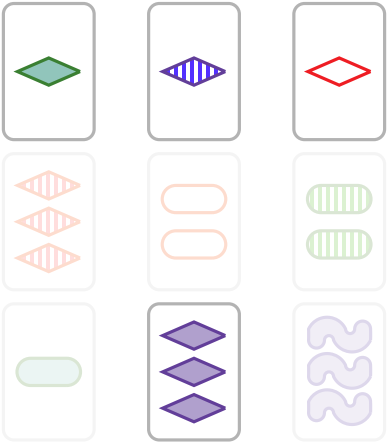
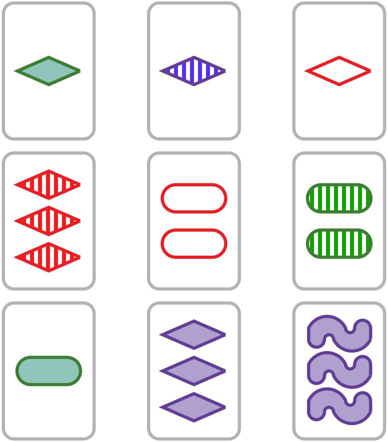


One way is to find a 1-SET and then add any card to it.
But that would be too easy...

The game of CAP

Can you find 4 cards in the same 2-SET that don't contain a 1-SET?



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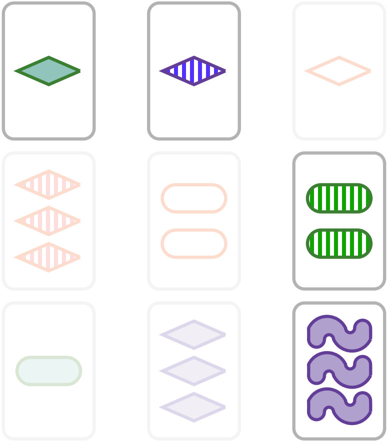
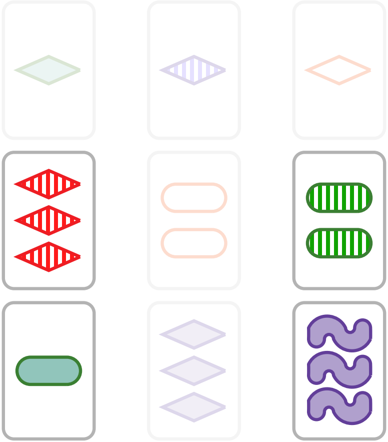


Here is one.

The game of CAP

Can you find 4 cards in the same 2-SET that don't contain a 1-SET?



And here is another.

Finding CAPs

How did I find these?

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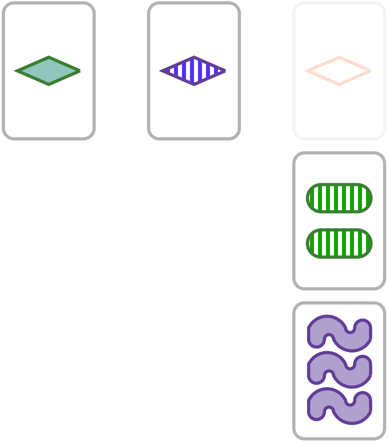

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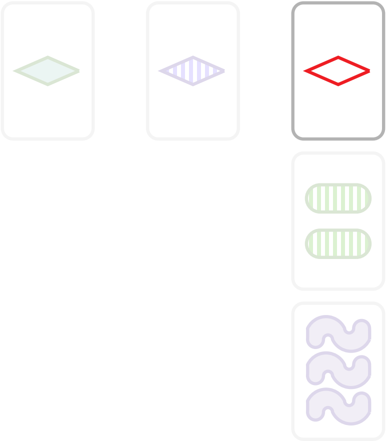

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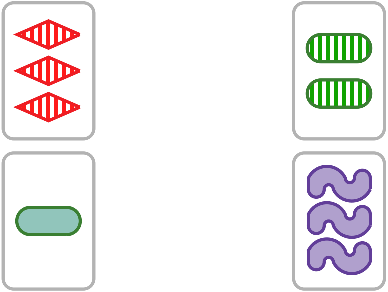

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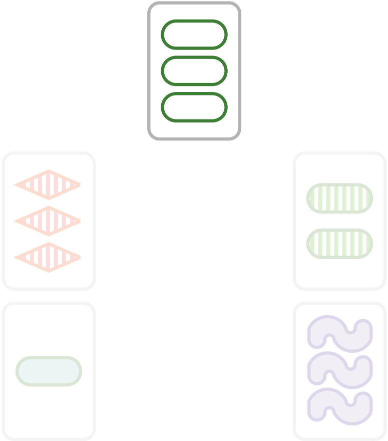

Geometry again

To understand why this works, it is helpful to think about the geometric analogy. One way to produce 4 points that lie on a plane, but no 3 of which lie on a line is to start with two lines that intersect at a point.

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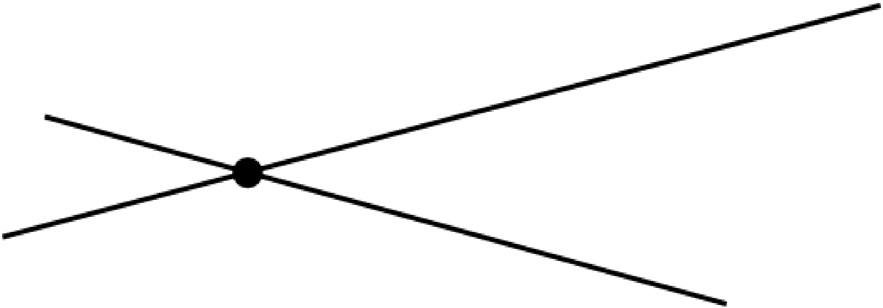

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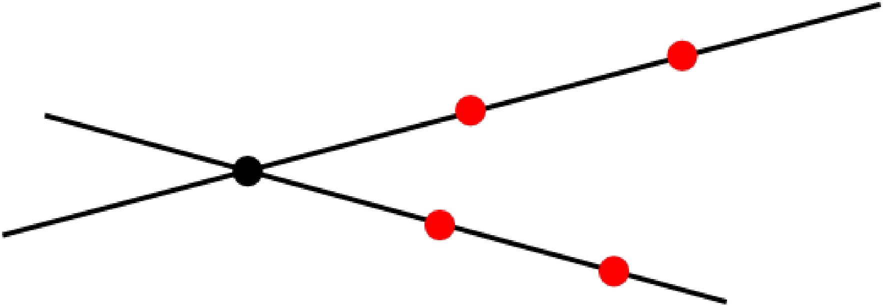

Then pick a pair of points on each line, making sure not to pick the intersection point.

Geometry again

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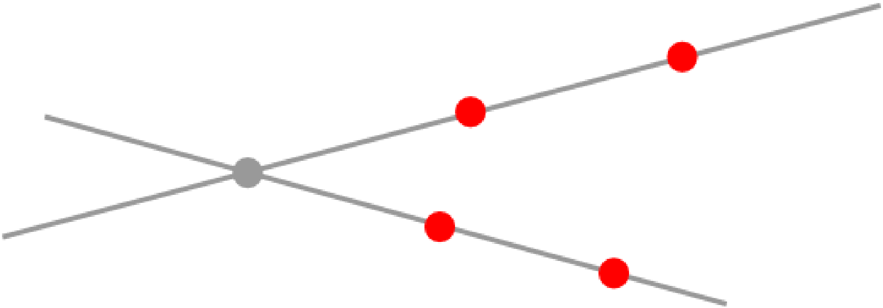

Now forget about the lines and you have 4 coplanar points, no 3 of which lie on a line!

But does every CAP arise this way?

Geometry suggests the answer is yes. Suppose we had four points on a plane:

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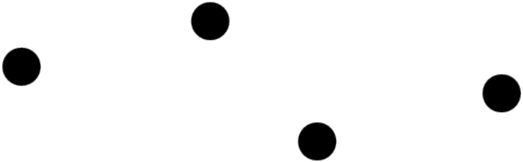

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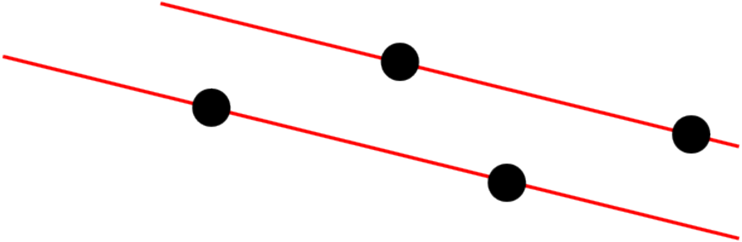

We could draw lines connecting them in pairs.

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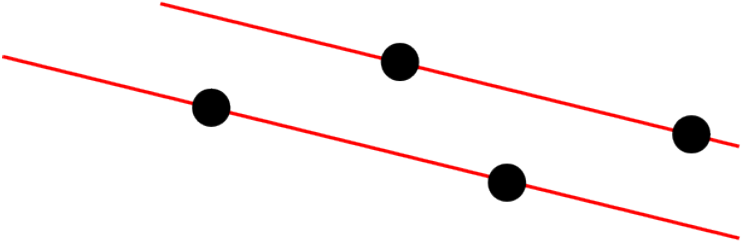

One of those pairs of lines might be parallel.

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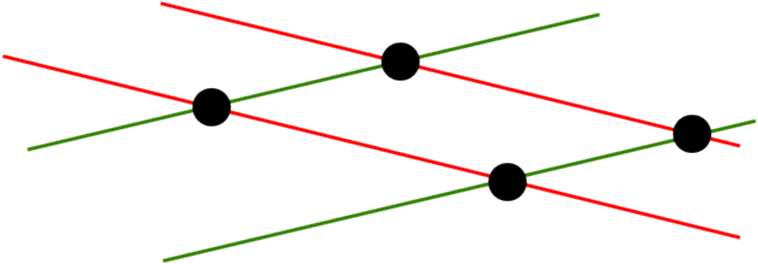

One of those pairs of lines might be parallel. Or even two.

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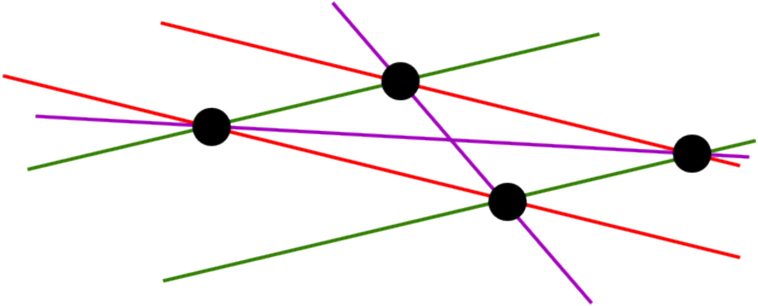

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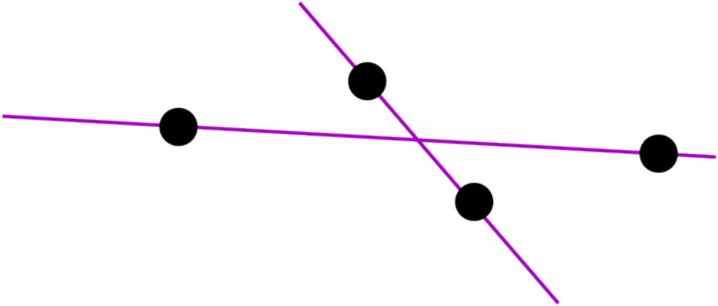

But not all three! There is always at least one pair of non-parallel lines in the plane that contains all four points.

Is this just an analogy?

Or is something deeper going on?

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Represent characteristics using numbers instead of pictures. Each card can be represented as a quadruple whose entries are 1, 2, and 3.

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one=1, two=2, three=3

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oval=1, diamond=2, squiggle=3

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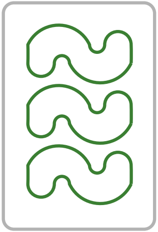
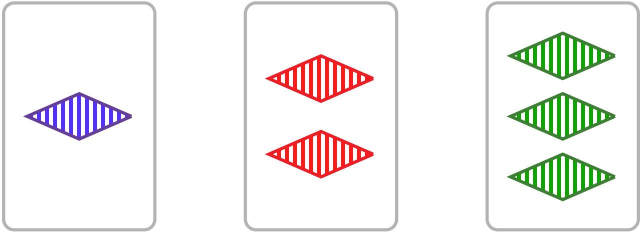
three=3, empty=1, green=2, squiggle=3



$= (3, 1, 2, 3)$

How to spot SETs numerically



$$A = (1, 2, 3, 1) \qquad B = (2, 2, 1, 1) \qquad C = (3, 2, 2, 1)$$

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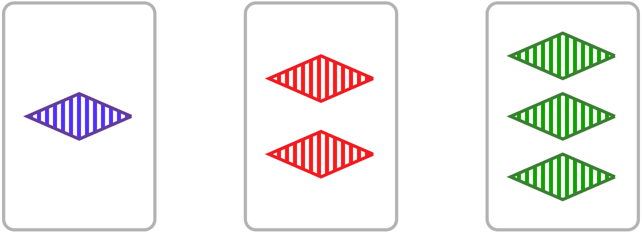

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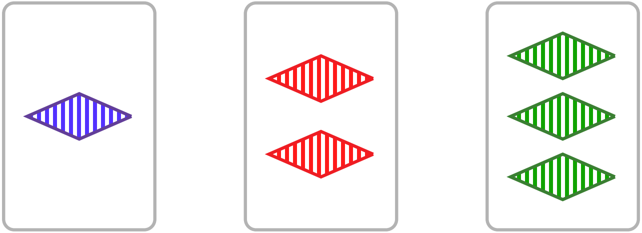

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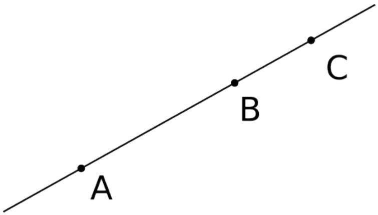
$$B - A = (1, 0, -2, 0) \equiv (1, 0, 1, 0) \bmod 3$$
$$C - B = (1, 0, 1, 0)$$

The points A, B, and C form a SET if and only if $C - B \equiv B - A \bmod 3$.

How to spot points on a line



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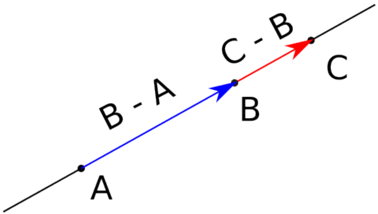

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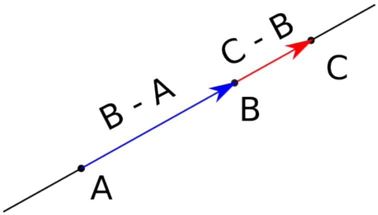

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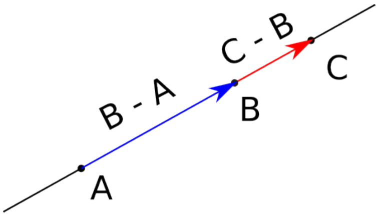

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The points A, B, and C lie on the same line if and only if $C - B = t(B - A)$ for some multiple t.

Fields

A *field* is a setting where one can add, subtract, multiply, and divide by nonzero elements. The rational numbers ($\mathbf{Q}$), the real numbers ($\mathbf{R}$), and the complex numbers ($\mathbf{C}$) are all fields. But there are many more examples...

The field with 3 elements

$\mathbf{F}_3$ consists of 3 elements: $\overline{0}$, $\overline{1}$, and $\overline{2}$.

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It works just like adding times on a clock. Or it would, if there were 6 hours in a day.

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Here is the equation for a line:

$$C - B = t(B - A)$$

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Here is the equation for a line in $\mathbf{F}_3$:

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The equation of a line is the same as the equation of a SET!

The game of SET geometrically

Find 3 distinct collinear points in $\mathbf{F}_3{}^4$.

The game of CAP geometrically

Find 4 coplanar points in ${\mathbf{F}_3}^4$ that are not collinear.

Affine caps

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The size a_n of the largest affine cap in $\mathbf{F}_3{}^n$ (from Davis and Maclagan):

n	a_n
1	2
2	4
3	9
4	20
5	45
6	$112 \leq a_6 \leq 114$
7	?
⋮	⋮