§8.1: Sequences

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Key Points:

- Think of a sequence as a comma-separated list:

$$a_1, a_2, a_3, \ldots, a_n, \ldots.$$

- A sequence is a function whose domain is the positive integers. You can graph a sequence of real numbers.
- We are often interested in the end behavior of a sequence, $\lim_{n\to\infty} a_n$. Hint: use the "connect the dots" function defined on $\mathbb{R}$ (i.e. $f(n) = a_n$). In particular,

$$\lim_{n\to\infty} a_n = \lim_{x\to\infty} f(x),$$

  and we can use Calc I tools like L'Hôpital's Rule.

- Some neat tools:
  - Squeeze Law (Sandwhich Theorem):
  - Showing an alternating sequence converges:
  - Showing an alternating sequence diverges:
  - Three ways to show a sequence is decreasing
    1.
    2.
    3.
  - Note: Bounded Monotonic (i.e. increasing or decreasing) sequences must converge.

Examples:

1. Consider the sequence $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots, a_n, \ldots$. Find a formula for a_n.

2. Consider the sequence $\frac{1}{3}, \frac{1}{6}, \frac{1}{11}, \frac{1}{18}, \ldots, a_n, \ldots$. Find a_n.

3. Consider the sequence $\frac{2}{3}, \frac{4}{9}, \frac{6}{27}, \frac{8}{81}, \ldots, a_n, \ldots$. Find a_n.

4. Consider the sequence $-\frac{5}{2}, \frac{8}{4}, -\frac{11}{8}, \frac{14}{16}, \ldots, a_n, \ldots$. Find a_n.

5. Consider the sequence $7, -\frac{9}{2}, \frac{11}{6}, -\frac{13}{24}, \ldots, a_n, \ldots$. Find a_n.

6. Suppose $a_n = \dfrac{\cos n}{n^2}$. Find $\lim\limits_{n\to\infty} a_n$.

7. Suppose $a_n = \dfrac{(-1)^n \ln n}{n}$. Find $\lim\limits_{n\to\infty} a_n$.

8. Suppose $a_n = \dfrac{(-1)^n \left(n^3+3\right)}{2n^3-1}$. Find $\lim\limits_{n\to\infty} a_n$.

9. Suppose $a_n = \dfrac{\sqrt{3n^2+4}}{n-1}$. Find $\lim\limits_{n\to\infty} a_n$.

10. Suppose $a_n = \left(1+\dfrac{1}{n}\right)^n$. Find $\lim\limits_{n\to\infty} a_n$.

11. Show $a_n = \dfrac{3^{n+2}}{5^n}$ is decreasing.

12. Show $a_n = \dfrac{n}{n+1}$ is increasing.

13. Show $a_n = \dfrac{n}{e^n}$ is decreasing.

14. Find a formula for a_n if $a_1 = 2$ and $a_{n+1} = a_n + 5$.

15. Find a formula for a_n if $a_1 = 4$ and $a_{n+1} = 3 \cdot a_n$.