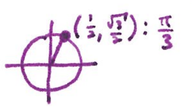
Math 2300-007: Quiz 2

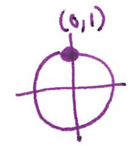
Name: Solutions : 1/25/18 Score: ________

1. (2 points) Evaluate the following

(a) $\arctan(\sqrt{3}) = \pi/3$ $\left(\frac{\sin(\frac{\pi}{3})}{\cos(\pi/3)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}\right)$



(c) $\sin\left(-\frac{3\pi}{2}\right) = 1$



(b) $\int x^{-1} + x^{-2}\, dx = \ln|x| - x^{-1} + C$
$= \ln|x| - \frac{1}{x} + C$

(d) $\frac{1 - \cos(2\theta)}{2} = \sin^2\theta$

2. (a) (2 points) Write down an integral that you can evaluate with integration by parts.

e.g. $\int xe^x dx$, $\int \ln x\, dx$, $\int \arctan(\frac{1}{x})dx$, $\int 3x^2 \cos x\, dx$, $\int e^x \cos x\, dx$, ...

(b) (3 points) Evaluate your integral from part (a).

Varies... Remember $\int u\,dv = uv - \int v\,du$.

- For $\int xe^x dx$, let $u = x$ $dv = e^x dx$
- For $\int \ln x\, dx$, let $u = \ln x$ $dv = dx$
- For $\int \arctan(\frac{1}{x})dx$, let $u = \arctan(\frac{1}{x})$ $dv = dx$ → Then simplify & u/du sub.
- For $\int 3x^2 \cos x\, dx$, let $u = 3x^2$ $dv = \cos x\, dx$ → By parts twice
- For $\int e^x \cos x\, dx$, let $u = e^x$ or $\cos x$ $dv = \cos x dx$ or $e^x dx$ → By parts twice & "boomerang"

3. (3 points) Integrate $\int \tan^3(x)\sec^4(x)\, dx$.

Method I:

$\int \tan^3(x)\sec^4(x)\, dx = \int \tan^3 x \sec^2 x \cdot \sec^2 x\, dx$

$= \int \tan^3 x\,(\tan^2 x + 1) \cdot \sec^2 x\, dx$

$\left\{ \begin{array}{l} u = \tan x \\ du = \sec^2 x\, dx \end{array} \right\}$ $= \int u^3 (u^2 + 1)\, du$

$= \int u^5 + u^3\, du$

$= \frac{1}{6}u^6 + \frac{1}{4}u^4 + C$

$= \frac{1}{6}\tan^6 x + \frac{1}{4}\tan^4 x + C$

Method II:

$\int \tan^3 x \sec^4 x\, dx = \int \tan^2 x \sec^3 x \cdot \tan x \sec x\, dx$

$= \int (\sec^2 x - 1)\sec^3 x \cdot \tan x \sec x\, dx$

$\left\{ \begin{array}{l} u = \sec x \\ du = \sec x \tan x \end{array} \right\}$ $= \int (u^2 - 1)u^3\, du$

$= \int u^5 - u^3\, du$

$= \frac{1}{6}u^6 - \frac{1}{4}u^4 + C$

$= \frac{1}{6}\sec^6 x - \frac{1}{4}\sec^4 x + C$