Math 2300–007: Exam II Study Guide

Topics List:

- §6.6: Applications of Integration
  - Work
    * Springs and Hooke's Law
    * Rope/chain lifting
    * Water pumping problems
  - Center of Mass
    * Moments M_x and M_y
  - Tips: Draw a small rectangle (dx or dy), draw a slice, and put a coordinate system on the picture.

- §8.1: Sequences
  - "Connect the dots" function
  - Limits involving ∞
  - L'Hôpital's Rule
  - Squeeze Theorem
  - Alternating sequences (look at the sequence $|a_n|$)
  - Tip: Identify the "dominant" function and factor it out

- §8.2: Intro to Series
  - Definition of a series
  - Partial sums (the sum of a series is the limit of the partial sums)
  - Divergence Test (if terms don't go to zero, the series diverges)
  - Telescoping series
  - Geometric series

- §8.3: Comparison Tests (need positive terms!)
  - Integral Test
  - p-series (if $p \leq 1$, $\sum \frac{1}{n^p}$ diverges, if $p > 1$, $\sum \frac{1}{n^p}$ converges)
  - Direct Comparison Test
  - Limit Comparison Test

- §8.4: Other Convergence Tests
  - Alternating series and Alternating Series Test
  - Absolute convergence
  - Ratio Test

Selected Example Problems Provided by Class for §6.6,8.1,8.2,8.3:

1. Find The work if a rope weighing 4lbs/ft hoists a 300lb weight up the side of a building 500 feet tall.

2. An inverted conical tank with height 20m and base diameter 25m contains oil with density 800kg/m^3. The height of the oil is 10m. How much work is done pumping all of the oil out of the top?
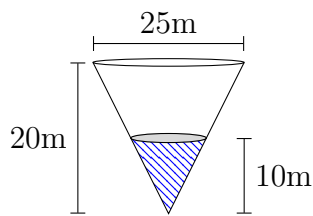
3. Problems 19 and 21 from page 473 of the course text. These are water pumping problems with pictures I couldn't easily replicate.

4. Sketch the region bounded by the curves, and visually estimate the location of the centroid. Then, find the exact coordinates of the centroid.

   (a) $y = 4 - x^2$, $y = 0$

   (b) $3x + 2y = 6$, $y = 0$, $x = 0$

5. Consider the sequence $-\frac{5}{2}, \frac{8}{4}, \frac{-11}{8}, \frac{14}{16}, \ldots, a_n, \ldots$. Find a_n. Does this sequence converge or diverge?

6. Suppose $a_n = \frac{\cos n}{n^2}$. Find $\lim\limits_{n \to \infty} a_n$.

7. Do the following series converge or diverge? How do you know? Can you figure out the exact sum?

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 3n + 1}{2n^2 + 4}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\cos n\right)$

   (c) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{2}{3}\right)^n$

   (d) $\displaystyle\sum_{n=0}^{\infty} 8\left(\frac{1}{3}\right)^n$

   (e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

(f) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

(g) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt[5]{n}}$

(h) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+4}}$

(i) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n^2}+1\right)^2$

(j) $\displaystyle\sum_{n=4}^{\infty}\frac{n^2}{n^3-3}$

(k) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt{2n^2+4n+1}}{n^3+9}$

(l) $\displaystyle\sum_{n=1}^{\infty}\frac{4n^2-n}{n^3+9}$

(m) $\displaystyle\sum_{n=0}^{\infty}\frac{2^n\sin^2(5n)}{4^n+\cos^2(n)}$

(n) $\displaystyle\sum_{n=3}^{\infty}\frac{e^{-n}}{n^2+2n}$

Some problems I added for §8.4:

1. Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

   (a) $\displaystyle\sum_{n=0}^{\infty}\frac{(-10)^n}{n!}$

   (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2^n}{n^4}$

   (c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{1+\ln(n)}$

2. Show that $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n5^n}$ is convergent. How many terms of the series do we need to add in order to find the sum to within 0.0001 of its actual value?

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Name of Test	How it works	Hypotheses	When to use	Example(s)
Divergence Test				
Geometric Series				
Integral Test				
Direct Comparison Test				
Limit Comparison Test				
Alternating Series Test				
Ratio Test				