§8.5: Power Series
(Created by Faan Tone Liu)

Key Points:

A. What is a **power series**?

- First Perspective: Inspired by polynomials, we create an "infinite-degree polynomial." For example:

- Second Perspective: Put a x^n as part of a series. For example:

- Third Perspective: A power series is a function where x is the input and the output is a series. For example:

A. Basic questions:

- For what x-values dos the power series converge? To answer this question, use the _____. The result is an interval called the **interval of convergence**. Important: Check the endpoints separately.

- To what value does the series converge?

Examples:

1. Consider the series $1 + x + x^2 + \cdots + x^n + \cdots$. For which values of x does the series converge?

2. Find the interval of convergence of the series $1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{3!} + \cdots$.

3. Find the interval of convergence of the series $1 - \dfrac{(x-3)}{2} + \dfrac{(x-3)^2}{4} - \dfrac{(x-3)^3}{8} + \cdots$.

4. Find the interval of convergence of the series $\displaystyle\sum_{n=0}^{\infty} n! x^n$.

5. Find the interval of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \dfrac{(2x+1)^n}{n3^n}$.