§8.4: Part I - Alternating Series

(Thanks to Faan Tone Liu)

Key Points:

- If the terms in a series alternate signs, we call the series an **alternating series**.
- An alternating series can be written in the form

$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n \quad \text{or} \quad \sum_{n=1}^{\infty}(-1)^n b_n, \quad \text{where } b_n \geq 0.$$

  (i.e. b_n includes no negative terms)

- **Alternating series test**:

- Note: Recall that to show b_n is decreasing, show

  __________ or __________ or __________

- Note: If in an alternating series, $\lim_{n\to\infty} b_n \neq 0$, then

- Alternating series remainder test: If $\sum(-1)^n b_n$ converges by the alternating series test, then

$$|\text{Error}| = |R_n| =$$

Examples:

1. (Alternating Harmonic Series) Does $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}$ converge or diverge?

2. Does $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}$ converge or diverge?

3. Does $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{e^n}$ converge or diverge?

4. Estimate $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^5}$ using three terms. How accurate is your estimate?

5. How many terms should we add to ensure that our estimate of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is within 0.0001 of the true value?