

INTEGRATION BY PARTS

1. LIATE

Once you have identified an integral as being on that can be best computed using integration by parts, you need to figure out what should be "u" and what should be "dv". As a general rule, remember the acronym "LIATE", and choose u in order of decreasing priority:

- Logarithmic
- Inverse Trigonometric
- Algebraic
- Trigonometric
- Exponential

Try using this in the following exercises:

Exercise 1.1. $\int x^3 \ln(x) dx$

Exercise 1.2. $\int \sin(x) \ln(\cos(x)) dx$

Exercise 1.3. $\int \arccos(x) dx$

2. ANOTHER GUIDELINE

As with any general heuristic as this one, "LIATE" is only helpful when we have an integrand which is a mix of algebraic and transcendental functions. But what about a function that is a product of algebraic functions? For instance, if $f(x) = x^3\sqrt{4-x^2}$, then we cannot apply LIATE in order to compute $\int f(x)dx$. However, try using the following yoga in this situation

"...Let dv be the most complicated part of the integrand that can be easily integrated, and let u be the portion of the integrand whose derivative is a simpler function..."

Try using this as a guideline when LIATE doesn't apply; try it out for the function $f(x)$ we used as an example above:

Exercise 2.1. $\int x^3\sqrt{4-x^2}dx$

What about a product of trigonometric functions, say $f(x) = \sin(x)\cos(x)$?

Exercise 2.2. $\int \cos(x)\sin(x)dx$

3. EXERCISES

Exercise 3.1. $\int x^3 \sqrt{9 - x^2} dx$

Exercise 3.2. $\int e^{2x} \sin(x) dx$

Exercise 3.3. $\int \frac{1}{x(\ln(x))^3} dx$

Exercise 3.4. $\int x \sin(x) \cos(x) dx$

Exercise 3.5. $\int x^2 \sqrt{x - 1} dx$