

Alternate statement of Taylor's inequality

Background information: If $f(x)$ is a function, and $T_n(x)$ is its n th-degree Taylor polynomial centered at a , then the **remainder** is the error in how the Taylor polynomial approximates the function. In other words, $R_n(x) = f(x) - T_n(x)$. (Note that if $T_n(x)$ is an underestimate, then $R_n(x)$ is positive, and if $T_n(x)$ is an overestimate, then $R_n(x)$ will be negative.) Of course we usually want $|R_n(x)|$ to be small, and Taylor's inequality gives a bound on how large that error can be. The theorem below uses M as an upper bound for the $(n + 1)$ st derivative of f , so usually the first step in error calculations is to figure out what to use for M .

Taylor's Inequality If $f^{(n+1)}$ is continuous and $|f^{(n+1)}| \leq M$ between a and x , then:

$$|R_n(x)| \leq \frac{M}{(n+1)!} |x - a|^{n+1}$$