

1. Evaluate the following integral:

$$\int \frac{x^2}{\sqrt{1-x^2}} dx$$

2. Compute the volume of the solid obtained by rotating the region enclosed by  $y = x$  and  $y = x^2$  around the  $y$ -axis.

3. Find the sum of each of the following series.

$$(a) \sum_{n=2}^{\infty} \frac{(-1)^n 5 \cdot 2^{n+1}}{e^n}$$

$$(b) \sum_{n=2}^{\infty} \left( \frac{1}{2n-1} - \frac{1}{2n+1} \right)$$

$$(c) \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)! 2^{2n+1}}$$

$$(d) \sum_{n=1}^{\infty} n \left( \frac{9}{10} \right)^{n+1}$$

4. As it happens,

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

If we estimate the sum by adding from  $n = 1$  to  $n = N$

$$\sum_{n=1}^N \frac{(-1)^{n+1}}{n^2}$$

what should  $N$  be to *guarantee* an approximation of  $\frac{\pi^2}{12}$  accurate to .0000005?

5. Find the mass  $m$  and center of mass  $(\bar{x}, \bar{y})$  of the plate enclosed by the  $x$ -axis and  $y = \ln x$  on  $[1, e]$  with constant density  $\rho$ .

6. Find the radius and interval of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{(x-10)^n}{n4^n}$$

7. Use any method to write down the Taylor series centered at  $a = 0$  for the function  $f(x) = x^2 e^{x^2}$ . Use your answer to determine  $f^{(8)}(0)$ .

8. You have succeeded John Hammond as facilitator of Jurassic Park.

Foolishly, you decided to again introduce velociraptors into the park. You clock one of the raptors as it starts at rest, accelerates to its top speed, and then immediately starts slowing down.

The raptor's speed is given by

$$v(t) = 100te^{-\frac{t}{2}},$$

where  $t$  is in seconds, and  $v(t)$  is in ft/s.

How long does the raptor take to hit its top speed? What is its top speed in ft/s? In mph?

What is the raptor's average speed over the interval  $[0, 15]$ ?