MIDTERM 3
CALCULUS 2

MATH 2300
FALL 2018

Monday, December 3, 2018
5:15 PM to 6:45 PM

Name	

PRACTICE EXAM

Please answer all of the questions, and show your work.
You must explain your answers to get credit.
You will be graded on the clarity of your exposition!

Date: October 27, 2018.

1
8 points

1. Match the following functions with their corresponding Maclaurin series:

(a) $e^{x^2/2} =$ ________________

(b) $\cos\left(\frac{x}{2}\right) =$ ________________

(c) $\frac{1}{(1-x)^2} =$ ________________

(d) $x \ \arctan(x) =$ ________________

(I) $\sum_{n=0}^{\infty} x^{2n}$

(II) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(2n)!}$

(III) $\sum_{n=1}^{\infty} n x^{n-1}$

(IV) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{2n+1}$

(V) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+1)!}$

(VI) $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$

2
12 points

2. Consider the power series $\sum_{n=1}^{\infty} \frac{(x-5)^n}{2^n n^2}$

2.(a). Find the *radius of convergence* of the power series. *Show all work in justifying your answer*.

2.(b). Find the *interval of convergence*. *Show all work in justifying your answer*.

3
12 points

3. Find the solution of the differential equation

$$y(x+1) + y' = 0$$

that satisfies the initial condition $y(-2) = 1$. Show all your work.

4
8 points

4. Given the following power series $\sum_{n=0}^{\infty} a_n(x-2)^n$ we know that at $x=0$ the series converges and at $x=8$ the series diverges. What do we know about the following values?

4.(a). At $x=3$ the series $\sum\limits_{n=0}^{\infty} a_n(x-2)^n$ is:

(i) Convergent
(ii) Divergent
(iii) We cannot determine its convergence/divergence with the given information.

4.(b). At $x=-4$ the series $\sum\limits_{n=0}^{\infty} a_n(x-2)^n$ is:

(i) Convergent
(ii) Divergent
(iii) We cannot determine its convergence/divergence with the given information.

4.(c). At $x=9$ the series $\sum\limits_{n=0}^{\infty} a_n(x-2)^n$ is:

(i) Convergent
(ii) Divergent
(iii) We cannot determine its convergence/divergence with the given information.

4.(d). The following series $\sum\limits_{n=0}^{\infty} a_n$ is:

(i) Convergent
(ii) Divergent
(iii) We cannot determine its convergence/divergence with the given information.

5
12 points

5.(a). Write the definition for the nth degree Taylor polynomial of a function $f(x)$ centered at $x = a$.

5.(b). Find the second degree Taylor polynomial for $f(x) = \ln(\sec(x))$ centered at $\pi/4$.

6
12 points

6.(a). Express the function $f(x) = \ln(1 + x^3)$ as a power series centered about $x = 0$.

6.(b). Express the definite integral $\int_0^1 \ln(1 + x^3)\, dx$ as an infinite series.

7
12 points

7.(a). Fill in the blanks to complete the statement of **Taylor's Inequality:**

If [ ] $\leq M$ on the interval between the center, a, and the point of approximation x, then the remainder, $R_n(x)$, of the nth degree Taylor polynomial $T_n(x)$, satisfies the inequality:

$$|R_n(x)| \leq \boxed{}$$

7.(b). Use Taylor's inequality to determine the number of terms of the Maclaurin series for e^x that should be used to estimate the number e with an error less than 0.6. Clearly justify your choice of M.

8
8 points

8. Each of the following slope fields represents one of the following differential equations. Match each slope field to the corresponding dfferential equation.

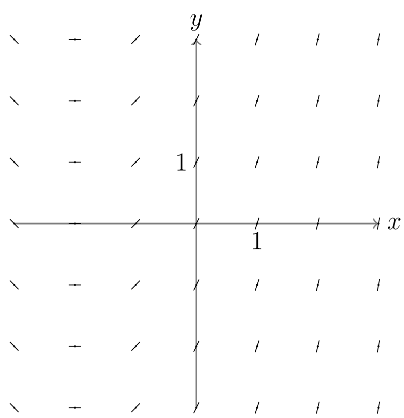
(a) $\dfrac{dy}{dx} = \dfrac{xy}{2}$ ________________

(b) $\dfrac{dy}{dx} = y - x - 2$ ________________

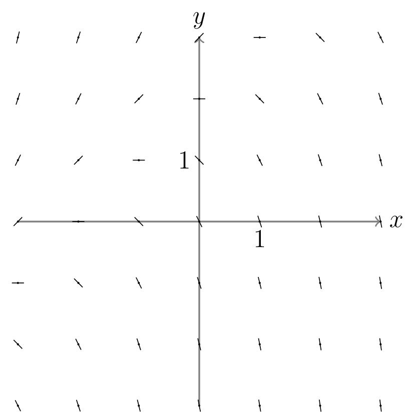
(c) $\dfrac{dy}{dx} = x + 2$ ________________

(d) $\dfrac{dy}{dx} = e^x$ ________________

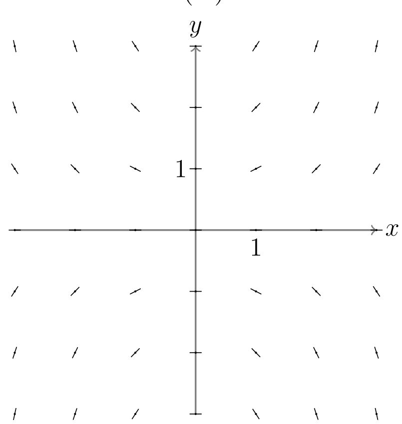
(I)



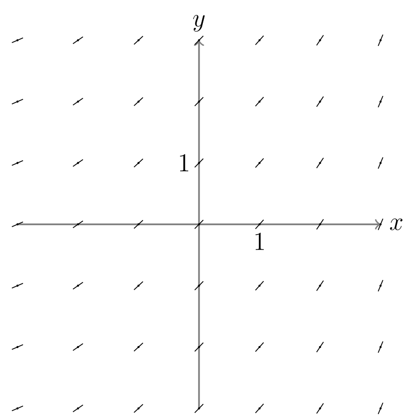
(III)



(II)



(IV)



9
6 points

9. Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$$

10
10 points

10. Assume we approximate the sum of the series

$$\sum_{n=1}^{\infty} \frac{2}{n^2}$$

by using the first 3 terms. Give an upper bound for the error involved in the approximation by using the Remainder Estimate for the Integral Test.