MIDTERM 2
CALCULUS 2

MATH 2300
FALL 2018

Monday, October 22,
5:15 PM to 6:45 PM.

Name | ____

PRACTICE EXAM

Please answer all of the questions, and show your work.
You must explain your answers to get credit.
You will be graded on the clarity of your exposition!

Date: October 22, 2018.

1
8 points

1. (4 points) Which of the following integrals gives the arc length of the function $f(x) = 3\cos(x)$ from $x = 0$ to $x = \pi/4$? You do not need to show any work for this problem.

(A). $\displaystyle\int_0^{\pi/4} \sqrt{1 + 9\cos^2(x)}\, dx$ (C). $\displaystyle\int_0^{\pi/4} \sqrt{1 - 3\sin(x)}\, dx$

(B). $\displaystyle\int_0^{\pi/4} \sqrt{1 + 9\sin^2(x)}\, dx$ (D). $\displaystyle\int_0^{\pi/4} \sqrt{1 - 9\sin^2(x)}\, dx$

2. (4 points) Which of the following integrals gives the average value of the function $g(x) = x\ln(x)$ between $x = 1$ and $x = 10$? You do not need to show any work for this problem.

(A). $\displaystyle\int_1^{10} x^2 \ln(x)\, dx$ (C). $\displaystyle\int_1^{10} \frac{x\ln(x)}{9}\, dx$

(B). $\displaystyle\int_1^{10} \frac{x\ln(x)}{10}\, dx$ (D). $\displaystyle\int_1^{10} x\ln(x)\, dx$

3
8 points

3. Consider the region bounded by the x-axis, the y-axis, the line $x = 5$, and the curve $y = xe^{-x}$.

3.(a). (4 points) Which of the following expressions gives the x-coordinate of the center of mass of the region? You do not need to show any work for this problem.

(A). $\dfrac{\int_0^5 xe^{-x}\,dx}{\int_0^5 e^{-x}\,dx}$ (C). $\dfrac{\int_0^5 x^2e^{-x}\,dx}{\int_0^5 xe^{-x}\,dx}$

(B). $\dfrac{\int_0^5 x^2e^{-2x}\,dx}{\int_0^5 2xe^{-x}\,dx}$ (D). $\dfrac{\int_0^5 xe^{-x}\,dx}{\int_0^5 x^2e^{-x}\,dx}$

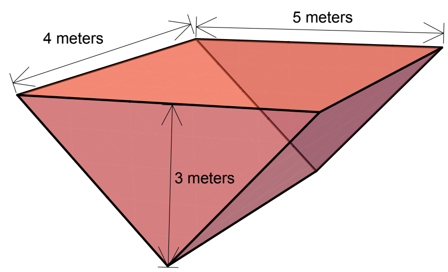
3.(b). (4 points) Which of the following epressions gives the y-coordinate of the center of mass of the region? You do not need to show any work for this problem.

(A). $\dfrac{\int_0^5 x^2e^{-2x}\,dx}{\int_0^5 2xe^{-x}\,dx}$ (C). $\dfrac{\int_0^5 x^2e^{-x}\,dx}{\int_0^5 xe^{-x}\,dx}$

(B). $\dfrac{\int_0^5 x^2e^{-x}\,dx}{\int_0^5 xe^{-x}\,dx}$ (D). $\dfrac{\int_0^5 x^2e^{-x}\,dx}{2\int_0^5 xe^{-x}}\,dx$

4
10 points

4. The solid pictured below shows a tank filled with water. Set up an integral that represents the work required to pump all of the water out of the the top of the tank. Use ρ for density of water and g for the acceleration due to gravity. Show any work that you use to arrive at your answer. *You do not need to evaluate the integral*.



5
20 points

5. Determine if each of the following series **converges conditionally, converges absolutely, or diverges**. Recall that a series is conditionally convergent if it converges but does not converge absolutely. Show all your work and carefully and fully justify your reasoning, including naming the convergence test.

5.(a). (10 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$

5.(b). (10 points) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

6
10 points

6. (10 points) Determine if the following series converges or diverges. Show all your work and carefully justify your reasoning. If the series converges, give the value of its sum.

$$\sum_{n=1}^{\infty} \frac{3 \cdot 2^{n+1}}{5^{n-1}}$$

7
10 points

7. (10 points) The infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

converges to some real number s. We want to estimate s by computing the partial sum

$$s_N = \sum_{n=1}^{N} \frac{(-1)^n}{n^2} = -1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \cdots + \frac{(-1)^N}{N^2}$$

How many terms should we use in the partial sum if we want the error to be less than or equal to $.0001 = 10^{-4}$?

8
24 points

8. (24 points) For each of the following series, determine if it converges absolutely, converges conditionally or diverges. Recall that a series is conditionally convergent if it converges but does not converge absolutely. You do not need to show any work.

8.(a). $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(A). converges absolutely (C). diverges
(B). converges conditionally

8.(b). $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

(A). converges absolutely (C). diverges
(B). converges conditionally

8.(c). $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{n^6}}$

(A). converges absolutely (C). diverges
(B). converges conditionally

8.(d). $\sum_{n=1}^{\infty} \frac{\arctan(n^2)}{n}$

(A). converges absolutely (C). diverges
(B). coverges conditionally

8.(e). $\sum_{n=1}^{\infty} \frac{1}{(-\pi)^n}$

(A). coverges absolutely (C). diverges
(B). converges conditionally

8.(f). $\sum_{n=1}^{\infty} \cos(\pi n)$

(A). converges absolutely (C). diverges
(B). converges conditionally

8.(g). $\sum_{n=1}^{\infty} \frac{(-n)^7}{2^n}$

(A). converges absolutely (C). diverges
(B). converges conditionally

8.(h). $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{5n+n^3}}{2n^2+3}$

(A). converges absolutely

(B). converges conditionally

(C). diverges

9
10 points

9. (10 points) Select the best method for determining whether the following series converge or diverge. You do not need to show any work.

9.(a). $\sum \frac{10}{n\ln(n)}$

(A). alternating series test
(B). divergence test
(C). integral test
(D). ratio test

9.(b). $\sum \frac{3^{2n-1}}{(2n+1)!}$

(A). p-series
(B). divergence test
(C). geometric series
(D). ratio test

9.(c). $\sum \frac{5n^3}{\sqrt{n^7-4}}$

(A). alternating series test
(B). divergence test
(C). ratio test
(D). limit comparison test

9.(d). $\sum \frac{(-1)^n\sqrt{2n^2+3}}{n-3}$

(A). alternating series test
(B). divergence test
(C). integral test
(D). ratio test

9.(e). $\sum n^{-1/2}$

(A). alternating series test
(B). divergence test
(C). p-series
(D). ratio test