1. **What is the Maclaurin series of** $f(x) = \dfrac{2}{(1+x)^3}$?

A) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n \frac{(n+1)(n+2)}{2}\, x^n$

B) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n\, (n+1)(n+2)\, x^n$ ✓

C) $\displaystyle\sum_{n=0}^{+\infty} (-1)^{n-1} \frac{(n+1)(n+2)}{2}\, x^n$

D) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n\, (n+1)(n+2)\, x^n$

E) $\displaystyle\sum_{n=0}^{+\infty} \frac{(n+1)(n+2)}{2}\, x^n$

2. **If the Maclaurin series of a fuction** $f(x)$ **is** $\displaystyle\sum_{n=1}^{+\infty} (-1)^n \frac{x^n}{3n(n+6)}$

then $f^{(6)}(0)$ **is equal to**

A) $\frac{5}{3}$ **B)** $\frac{5}{2}$ **C)** $\frac{10}{3}$ ✓ **D)** $\frac{9}{7}$ **E)** $\frac{8}{5}$

3. **Find the interval of convergence of** $\displaystyle\sum_{n=1}^{+\infty} \frac{(-1)^n 3^n}{n\sqrt{n}}\, x^n$

A) $[0, 1/3]$

B) $(-1/3, 1/3)$

C) $[-1/3, 1/3)$

D) $(-1/3, 1/3]$

E) $[-1/3, 1/3]$ ✓

4. **Calculate the first non-zero term of the Maclaurin series of** $f(x) = \ln(\sec x)$

A) $\dfrac{x^2}{2}$ ✓ **B)** $-\dfrac{x^2}{2}$ **C)** x^2 **D)** $-x^2$ **E)** $\dfrac{x^3}{6}$

5. **Knowing that the Maclaurin series of $\ln(1+x)$ is given by**

$$\ln(1+x) = \sum_{n=1}^{+\infty} (-1)^{n-1} \frac{x^n}{n}$$

find the smallest number of terms of the series that one needs to add to compute $\ln(1.1)$ with an error less than or equal to 10^{-8}.

A) 8 **B)** 3 **C)** 5 **D)** 9 **E)** 7 ✓

6. **The Maclaurin series for $f(x) = \dfrac{x}{(1+x^2)^2}$ is:**

A) $\sum_{n=1}^{\infty} (-1)^n x^{2n}$

B) $\sum_{n=1}^{\infty} (-1)^n 2n x^{2n-1}$

C) $\sum_{n=1}^{\infty} (-1)^n n x^{2n-1}$

D) $\sum_{n=1}^{\infty} (-1)^{n+1} n x^{2n-1}$ ✓

E) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$

7. **Find the first three terms of the Taylor series for $f(x) = \cos x$ about $a = \frac{\pi}{3}$,**

A) $\frac{1}{2} - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) - \frac{1}{4}\left(x - \frac{\pi}{3}\right)^2$ ✓

B) $\frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) + \frac{1}{4}\left(x - \frac{\pi}{3}\right)^2$

C) $\frac{1}{2} - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) - \frac{1}{2}\left(x - \frac{\pi}{3}\right)^2$

D) $\frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) - \frac{1}{4}\left(x - \frac{\pi}{3}\right)^2$

E) $\frac{1}{2} - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right) + \frac{1}{2}\left(x - \frac{\pi}{3}\right)^2$

8. **Use the first two non-zero terms of the Maclaurin series of $\ln(\cos x)$ to estimate $\int_0^1 \ln(\cos x)\,dx$**

A) $\frac{1}{5}$ **B)** $-\frac{1}{5}$ **C)** $\frac{1}{6}$ **D)** $\frac{11}{60}$ **E)** $-\frac{11}{60}$ ✓

9. **If we compute the sum of the fewest terms necessary to guarantee that the error is less than 0.05, using Estimation Theorem for Alternating Series, then what is the estimate for e^{-1}?**

A) $\frac{11}{8}$ **B)** $\frac{3}{8}$ **C)** $\frac{3}{7}$ **D)** $\frac{2}{5}$ **E)** $\frac{1}{3}$ ✓

10. **Suppose that the series $\sum_{n=1}^{+\infty} c_n(x-3)^n$ converges when $x = 1$ and diverges when $x = 7$.**

From the above information, which of the following statements can we conclude to be true?

I. **The radius of convergence is $R \geq 2$.**

II. **The power series converges at $x = 4.5$**

III. **The power series diverges at $x = 6.5$**

A) I and II only ✓

B) I and III only

C) II and III only

D) All of them

E) None of them

11. **Find the coefficient of x^6 in the power series expansion of** $\dfrac{2}{1+2x^2}$

A) 8 **B)** -8 **C)** 32 **D)** -16 ✓ **E)** -64

12. **The power series representation (centered at $a = 0$) and the interval of convergence for $f(x) = \ln(4 - x^2)$ are:**

A) $-\sum_{n=0}^{+\infty} \dfrac{x^{2n+2}}{(2n+2)4^{n+1}} \quad I = (-2, 2)$

B) $-2\sum_{n=0}^{+\infty} \dfrac{x^{2n+2}}{(2n+2)4^{n+1}} \quad I = (-2, 2)$

C) $-2\sum_{n=0}^{+\infty} \dfrac{x^{2n+2}}{(2n+2)4^{n+1}} + \ln 4 \quad I = (-2, 2)$ ✓

D) $-2\sum_{n=0}^{+\infty} \dfrac{x^{2n+2}}{(2n+2)4^{n+1}} + \ln 4 \quad I = [-2, 2)$

E) $-\dfrac{1}{2}\sum_{n=0}^{+\infty} \dfrac{x^{2n+2}}{(2n+2)4^{n+1}} + \ln 4 \quad I = (-2, 2)$

13. **Using Maclaurin series and EstimationTheorem for alternating series, we can obtain the approximation**

$$\int_0^{0.1} \frac{1}{1+x^2}\,dx \approx 0.1 - \frac{(0.1)^3}{3} \text{ with error } \leq c$$

The value of c is

A) $(0.1)^3$ **B)** $(0.1)^5$ **C)** $(0.1)^7$ **D)** $\frac{(0.1)^3}{3!}$ **E)** $\frac{(0.1)^5}{5}$ ✓

14. **Find the coefficient of x^5 in the power series expansion of** $\frac{x^2+1}{x-2}$

A) $-\frac{1}{64}$ **B)** $\frac{3}{64}$ **C)** $-\frac{3}{64}$ **D)** $\frac{5}{64}$ **E)** $-\frac{5}{64}$ ✓

15. **Find the interval of convergence for the Taylor series** $\sum_{n=0}^{+\infty} \frac{3^n}{n^n}(x-5)^n$

A) $\left(-\frac{1}{3}, \frac{1}{3}\right)$ **B)** $\left(\frac{14}{3}, \frac{16}{3}\right)$ **C)** $\left(\frac{15-e}{3}, \frac{15+e}{3}\right)$ **D)** $\left(\frac{15-e}{3}, \frac{15+e}{3}\right]$ **E)** $(-\infty, \infty)$ ✓

16. **Which of the following is the interval of convergence of the power series** $\sum_{n=1}^{+\infty} (-1)^n \frac{n^2(x-2)^n}{3^n(n^3+2)}$

A) $(0, 6)$ **B)** $[0, 6)$ **C)** $(-1, 5]$ ✓ **D)** $[-1, 5)$ **E)** $[-1, 5]$

17. **Let $f(x)$ be the function which is represented by the power series**

$$f(x) = \sum_{n=1}^{+\infty} (-1)^n \frac{(x-1)^n}{n^3}$$

The fifth derivative of f at $x = 1$ is

A) $\frac{1}{2}$ **B)** $-\frac{37}{81}$ **C)** $-\frac{24}{25}$ ✓ **D)** $\frac{25}{96}$ **E)** $\frac{1}{4}$

18. **Find the coefficient of x^4 of the Maclaurin series of $f(x) = \sqrt{1+x}$**

A) $\frac{1}{57}$ **B)** $-\frac{75}{128}$ **C)** $-\frac{5}{128}$ ✓ **D)** $\frac{8}{57}$ **E)** $\frac{9}{77}$

19. **Find the Taylor series of $f(x) = \dfrac{1}{5-x}$ centered at $a = 1$**

A) $\displaystyle\sum_{n=0}^{+\infty} \frac{(x-1)^n}{5^n}$

B) $\displaystyle\sum_{n=0}^{+\infty} \frac{(x-1)^n}{5^{n+1}}$

C) $\displaystyle\sum_{n=0}^{+\infty} \frac{(x-1)^n}{5^n n!}$

D) $\displaystyle\sum_{n=0}^{+\infty} \frac{(x-1)^n}{4^{n+1}}$ ✓

E) $\displaystyle\sum_{n=0}^{+\infty} \frac{(x-1)^n}{4^n}$

20. **Find the Maclaurin series of $\displaystyle\int x^2 \sin x \, dx$**

A) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+3}}{(2n+3)!}$

B) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+3}}{(2n+1)!}$

C) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+3}}{(2n+3)(2n+1)!}$

D) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+4}}{(2n+4)(2n+1)!}$ ✓

E) $\displaystyle\sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+4}}{(2n+4)!(2n+1)!}$

21. **Find the Maclaurin series of** $f(x) = \dfrac{1}{(1-x)^4}$

A) $\displaystyle\sum_{n=3}^{+\infty} (-1)^n \frac{n(n-1)(n-2)}{6}\, x^{n-3}$

B) $\displaystyle\sum_{n=3}^{+\infty} \frac{n(n-1)(n-2)}{6}\, x^{n-3}$ ✓

C) $\displaystyle\sum_{n=2}^{+\infty} (-1)^n n(n-1) x^{n-2}$

D) $\displaystyle\sum_{n=2}^{+\infty} \frac{x^{n-2}}{n(n-1)}$

E) $\displaystyle\sum_{n=2}^{+\infty} \frac{x^{n-2}}{2n(n-1)}$

22. **Use a Taylor polynomial to approximate** $\displaystyle\int_0^1 e^{-x^3}\, dx$ **with error less than** 0.01**. The smallest number of terms that are needed for this accuracy is**

A) 2 **B)** 3 ✓ **C)** 4 **D)** 5 **E)** 6

23. **Determine the sum of the series** $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+3)(2n+1)}$

A) $\frac{\pi}{2}$ **B)** $\frac{\pi-2}{4}$ ✓ **C)** $\frac{\pi-1}{4}$ **D)** $\frac{\pi-4}{4}$ **E)** $\frac{\pi}{6}$

24. **The first** 4 **nonzero terms in the Maclaurin series of** $f(x) = (4+x)^{3/2}$ **are:**

A) $8 + 3x - \frac{3x^2}{8} + \frac{x^3}{16}$

B) $8 + 3x + \frac{3x^2}{16} - \frac{x^3}{128}$ ✓

C) $1 + \frac{3x}{2} + \frac{3x^2}{4} - \frac{3x^3}{8}$

D) $1 + \frac{3x}{2} + \frac{3x^2}{8} - \frac{x^3}{8}$

E) $1 + \frac{3x}{2} - \frac{3x^2}{16} + \frac{x^3}{64}$

25. **Suppose that the power series**

$$\sum_{n=0}^{\infty} c_n(x-5)^n$$

converges when $x = 2$ and diverges when $x = 10$.

From the above information, which of the following statements can we conclude to be true?

I: The radius of convergence R satisfies $3 \le R \le 5$.

II: We can NOT determine the interval of convergence from the above information only.

III: The derivative of the power series is $\sum_{n=1}^{\infty} nc_n(x-5)^{n-1}$, which converges when $x = 3$.

A) I and II only

B) I and III only

C) II and III only

D) All of them ✓

E) None of them