1. (4 points) Which of the following integrals gives the arc length of the function $f(x) = 3\cos(x)$ from $x = 0$ to $x = \pi/4$? You do not need to show any work for this problem.

(A) $\int_0^{\pi/4} \sqrt{1 + 9\cos^2(x)}\, dx$

(B) $\boxed{\int_0^{\pi/4} \sqrt{1 + 9\sin^2(x)}\, dx}$

(C) $\int_0^{\pi/4} \sqrt{1 - 3\sin(x)}\, dx$

(D) $\int_0^{\pi/4} \sqrt{1 - 9\sin^2(x)}\, dx$

2. (4 points) Which of the following integrals gives the average value of the function $g(x) = x\ln(x)$ between $x = 1$ and $x = 10$? You do not need to show any work for this problem.

(A) $\int_1^{10} x^2 \ln(x)\, dx$

(B) $\int_1^{10} \frac{x\ln(x)}{10}\, dx$

(C) $\boxed{\int_1^{10} \frac{x\ln(x)}{9}\, dx}$

(D) $\int_1^{10} x\ln(x)\, dx$

3. Consider the region bounded by the $x$-axis, the $y$-axis, the line $x = 5$, and the curve $y = xe^{-x}$.

(a) (4 points) Which of the following expressions gives the $x$-coordinate of the center of mass of the region? You do not need to show any work for this problem.

(A) $\frac{\int_0^5 xe^{-x}\, dx}{\int_0^5 e^{-x}\, dx}$

(B) $\frac{\int_0^5 x^2e^{-2x}\, dx}{\int_0^5 2xe^{-x}\, dx}$

(C) $\boxed{\frac{\int_0^5 x^2e^{-x}\, dx}{\int_0^5 xe^{-x}\, dx}}$

(D) $\frac{\int_0^5 xe^{-x}\, dx}{\int_0^5 x^2e^{-x}\, dx}$

(b) (4 points) Which of the following expressions gives the $y$-coordinate of the center of mass of the region? You do not need to show any work for this problem.

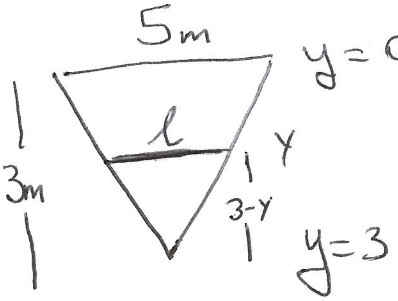
(A) $\boxed{\frac{\int_0^5 x^2e^{-2x}\, dx}{\int_0^5 2xe^{-x}\, dx}}$

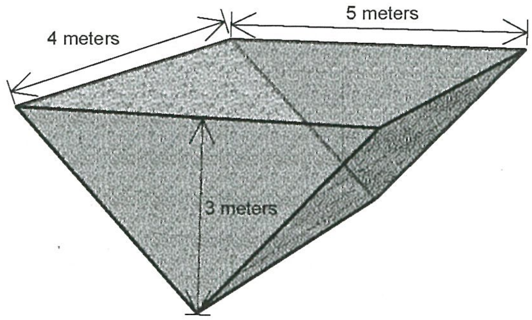
(B) $\frac{\int_0^5 x^2e^{-x}\, dx}{\int_0^5 xe^{-x}\, dx}$

(C) $\frac{\int_0^5 xe^{-2x}\, dx}{\int_0^5 xe^{-x}\, dx}$

(D) $\frac{\int_0^5 x^2e^{-x}\, dx}{2\int_0^5 xe^{-x}\, dx}$

4. (10 points) The solid pictured below shows a tank filled with water. Set up an integral that represents the work required to pump all of the water out of the the top of the tank. Use $\rho$ for density of water and $g$ for the acceleration due to gravity. Show any work that you use to arrive at your answer. You do not need to evaluate the integral.





$$\frac{l}{3-y} = \frac{5}{3} \quad l = \frac{5}{3}(3-y) \quad \text{Area} = 4 \cdot \frac{5}{3}(3-y) = \frac{20}{3}(3-y)\, m$$

$$\text{Volume} \approx \frac{20}{3}(3-y)\Delta y \; m^3$$

$$\text{mass} \approx \rho \frac{kg}{m^3} \cdot \frac{20}{3}(3-y)\Delta y \, m^3 = \frac{20}{3}\rho(3-y)\Delta y \; kg$$

$$\Delta F \approx g \cdot \text{mass} = \frac{20}{3}\rho g(3-y)\Delta y \; N$$

$$\Delta W \approx \text{force} \cdot \text{distance} = \frac{20}{3}\rho g(3-y)y\Delta y \; J$$

$$\int_{\boxed{0}}^{\boxed{3}} \boxed{\frac{20}{3}\rho g(3-y)y} \, d\boxed{y}$$

5. Determine if each of the following series **converges conditionally, converges absolutely, or diverges**. Recall that a series is conditionally convergent if it converges, but does not converge absolutely. Show all your work and carefully and fully justify your reasoning, including naming the convergence test(s) you are using.

(a) (10 points) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+2}}$ use Limit Comparison Test $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n+2}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+2}} = \lim_{n\to\infty} \sqrt{\frac{n}{n+2}} =$$

$$\sqrt{\lim_{n\to\infty} \frac{n}{n+2}} = \sqrt{1} = 1$$

Since $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ Diverges $= \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

So does $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+2}}$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$ use Alternating series Test

$\lim_{n\to\infty} \frac{1}{\sqrt{n+2}} = 0$ and $\left\{\frac{1}{\sqrt{n+2}}\right\}$ is a decreasing sequence because the numerator is constant and the denominator is increasing.

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$ converges by AST

Hence $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$ converges conditionally.

Ratio Test

(b) (10 points) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

$$\lim_{n\to\infty} \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} = \lim_{n\to\infty} \frac{\cancel{2^n}\cdot 2}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{2^n}} = \lim_{n\to\infty} \frac{2}{n+1} = 0$$

$\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converges Absolutely by the Ratio Test

6. (10 points) Determine if the following series converges or diverges. Show all your work and carefully justify your reasoning. If the series converges, give the value of its sum.

$\sum_{n=1}^{\infty} \frac{3 \cdot 2^{n+1}}{5^{n-1}}$.

$$= \sum_{n=1}^{\infty} \frac{3\cdot 2^n \cdot 2}{5^n \cdot 5^{-1}} = \sum_{n=1}^{\infty} 30\left(\frac{2}{5}\right)^n =$$

$$\sum_{n=1}^{\infty} 30\cdot\frac{2}{5}\left(\frac{2}{5}\right)^{n-1} = \sum_{n=1}^{\infty} 12\left(\frac{2}{5}\right)^{n-1}$$

$-1 < r = 2/5 < 1$ so this geometric series converges

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \quad \text{for } -1<r<1$$

converges to $\frac{12}{1-2/5} = \frac{12}{3/5} = 20$

7. (10 points) The infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

converges to some real number $s$. We want to estimate $s$ by computing the partial sum

$$s_n = \sum_{k=1}^{n} \frac{(-1)^k}{k^2} = -1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \cdots + \frac{(-1)^n}{n^2}$$

How many terms must the partial sum contain if we want to guarantee that the error is less than or equal to $.0001 = 10^{-4}$?

$$|S - S_n| < \frac{1}{(n+1)^2} = \frac{1}{10^4}$$

$$(n+1)^2 = 10^4$$

$$n+1 = 10^2 = 100$$

$$n = 99$$

Partial sum must contain at least 99 terms.

8. (24 points) For each of the following series, determine if it converges absolutely, converges conditionally or diverges. Recall that a series is conditionally convergent if it converges, but does not converge absolutely. You do not need to show any of your work.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(b) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{n^6}}$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan(n^2)}{n}$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(-\pi)^n}$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(f) $\displaystyle\sum_{n=1}^{\infty} \cos(\pi n)$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{(-n)^7}{2^n}$

(A) converges absolutely

(B) converges conditionally

(C) diverges

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{5n+n^3}}{2n^2+3}$

(A) converges absolutely

(B) converges conditionally

(C) diverges

9. (10 points) Select the best method for determining whether the following series converge or diverge. You do not need to show any of your work.

(a) $\sum \frac{10}{n \ln(n)}$

(A) alternating series test

(B) divergence test

(C) integral test

(D) ratio test

(b) $\sum \frac{3^{2n-1}}{(2n+1)!}$

(A) p-series

(B) divergence test

(C) geometric series

(D) ratio test

(c) $\sum \frac{5n^3}{\sqrt{n^7 - 4}}$

(A) alternating series test

(B) divergence test

(C) ratio test

(D) limit comparison test

(d) $\sum \frac{(-1)^n \sqrt{2n^2+3}}{n-3}$

(A) alternating series test

(B) divergence test

(C) integral test

(D) ratio test

(e) $\sum n^{-1/2}$

(A) alternating series test

(B) divergence test

(C) p-series

(D) ratio test